

# Impact Parameter Dependent Parton Distributions and Off-Forward Parton Distributions for $\zeta \rightarrow 0$

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It is shown that off-forward parton distributions for  $\zeta = 0$ , i.e. where the initial and final state differ only in their transverse momenta, can be interpreted in terms of a simultaneous measurement of the longitudinal momentum and transverse position (impact parameter) of partons in the infinite momentum frame.

## I. INTRODUCTION

Deeply virtual Compton scattering (DVCS) provides a novel tool to explore hadron structure. In contrast to deep-inelastic scattering (DIS), where one measures the imaginary part of the forward Compton amplitude only, DVCS allows measuring the off-forward Compton amplitude. From the parton point of view this implies that DVCS allows measuring off-forward matrix elements of parton correlation functions, i.e. on can access light-cone correlation functions of the form [1,2].<sup>1</sup>

$$f_\zeta(x, t) \equiv \int \frac{dx^-}{4\pi} \langle p' | \bar{\psi}(0) \gamma^+ \psi(x^-) | p \rangle e^{ixp^+ x^-}, \quad (1.1)$$

where  $x^\pm = x^0 \pm x^3$  and  $p^\pm = p^0 \pm p^3$  refers to the usual light-cone components and  $t \equiv q^2 = (p - p')^2$  is the invariant momentum transfer. The “off-forwardness” (or skewedness) in Eq. (1.1) is defined to be  $\zeta \equiv \frac{q^-}{p^+}$ . From the point of view of parton physics in the infinite momentum frame, these off-forward parton distributions (OFPDs) have the physical meaning of the amplitude for the process that a quark is taken out of the nucleon with momentum fraction  $x$  and then it is inserted back into the nucleon with a four momentum transfer  $q^\mu$  [3]. It was immediately recognized that off-forward parton distributions play a dual roles in that they combine features of both form factors and conventional parton distribution functions [1,2]: for  $\zeta = t = 0$  one recovers conventional parton distributions, i.e. momentum distributions in the infinite momentum frame (IMF), while when one integrates  $f_\zeta(x, t)$  over  $x$ , one obtains a form factor, i.e. the Fourier transform of a coordinate space density (in the Breit frame!).

However, the physical interpretation of the general case still remained obscure, mainly because the initial and final state in Eq. (1.1) are not the same and therefore, in general,  $f_\zeta(x, t)$  cannot be interpreted as a ‘density’ but rather their physical significance is that of a probability amplitude.

In this note, we will study a more general limiting case, namely  $\zeta = 0$ , but  $t \neq 0$ , and we will argue that

$$f(x, t) \equiv f_{\zeta=0}(x, t) \quad (1.2)$$

has a simple interpretation in terms of a density as well, namely as the Fourier transform of the light-cone momentum/impact parameter density w.r.t. the impact parameter.

In a light-front (LF) framework it is easy to see that the case  $\zeta = 0$  is particularly simple since there only terms diagonal in the Fock space contribute to  $f(x, t)$  (just as it is the case for ordinary parton distributions). Explicit Fock space representations for  $f(x, t)$  can be found in Refs. [4,5]. Making a Gaussian ansatz for the Fock space components, it is therefore straightforward to see the connection between the  $t$ -dependence of  $f(x, t)$  and the Gaussian size parameter [5,6]. In this paper, we will demonstrate that this very intuitive connection is valid independent of specific models and that it is in fact possible to determine parton distributions as a function of the impact parameter, provided  $f(x, t)$  is known.

Of course,  $\zeta = 0$  with  $t \neq 0$  cannot be achieved in virtual Compton scattering at finite energies because it always takes some longitudinal momentum transfer in order to convert a virtual photon into a real photon, i.e. strictly speaking  $\zeta = 0$  would correspond to real (wide angle) Compton scattering [6,7]. However, as a limiting case ( $\zeta \rightarrow 0$ ),  $f(x, t)$  is relevant for DVCS as well.

The paper is organized as follows. In Section II, we use a familiar observable, the elastic charge form factor, to illustrate how relativistic effects may spoil the identification of Fourier transforms of position space distributions with form factors. Since this is a well known phenomenon for form factors, Section II will mainly serve to introduce our notation and reasoning. In Section III, we will then generalize the results from Section II to  $\zeta = 0$  OFPDs. Section III contains the derivation of the main result of this paper, namely the identification of Fourier transforms of impact parameter dependent parton distributions with OFPDs. The results are summarized in Section IV.

<sup>1</sup>For some history on DVCS see Ref. [3].

## II. FORM FACTORS AND CHARGE DISTRIBUTIONS

Nonrelativistic intuition suggests to interpret ordinary charge form factors as Fourier transforms of charge distributions in position space. As a warmup exercise, we will in the following reexamine the limitations of this interpretation in a relativistic framework.

Since the charge distribution of a plane wave is ill defined, it is useful to start from a wave packet <sup>2</sup>

$$|\Psi\rangle = \int \frac{d^3p}{\sqrt{2E_{\vec{p}}(2\pi)^3}} \psi(\vec{p}) |\vec{p}\rangle, \quad (2.1)$$

where  $E_{\vec{p}} = \sqrt{M^2 + \vec{p}^2}$ . Momentum space eigenstates are normalized covariantly as usual, i.e.  $\langle \vec{p}' | \vec{p} \rangle = 2E_{\vec{p}} \delta(\vec{p}' - \vec{p})$ . Using the usual definition of the charge form factor

$$\langle \vec{p}' | \rho(\vec{0}) | \vec{p} \rangle = (E_{\vec{p}'} + E_{\vec{p}}) F(q^2), \quad (2.2)$$

where

$$q^2 = (E_{\vec{p}'} - E_{\vec{p}})^2 - \vec{q}^2 \quad (2.3)$$

and  $\vec{q} = \vec{p}' - \vec{p}$ , one obtains for the Fourier transform of the charge distribution in the wave packet

$$\begin{aligned} \mathcal{F}_\psi(\vec{q}) &\equiv \int d^3x e^{-i\vec{q}\cdot\vec{x}} \langle \Psi | \rho(\vec{x}) | \Psi \rangle \\ &= \int \frac{d^3p}{\sqrt{2E_{\vec{p}}2E_{\vec{p}'}}} \Psi^*(\vec{p} + \vec{q}) \Psi(\vec{p}) \langle \vec{p}' | \rho(\vec{0}) | \vec{p} \rangle \\ &= \frac{1}{2} \int d^3p \frac{E_{\vec{p}} + E_{\vec{p}'}}{\sqrt{E_{\vec{p}}E_{\vec{p}'}}} \Psi^*(\vec{p} + \vec{q}) \Psi(\vec{p}) F(q^2). \end{aligned} \quad (2.4)$$

Note that  $q^2$  still depends implicitly on  $\vec{p}$  (2.3) and thus in general one cannot pull  $F(q^2)$  out of the integral! Eq. (2.4) clearly illustrates how the charge distribution in the wave packet is obtained from a convolution of the Form factor with the spatial distribution of the wave packet as well as various relativistic effects.

Initially, the wave packet was only introduced in order to be able to cleanly define a charge distribution. On the other hand, we are interested only in the intrinsic charge distribution of the hadron, i.e. not in the distribution due to the wave packet and therefore we would like to get rid of anything associated with the wave packet in Eq. (2.4).

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<sup>2</sup>I would like to thank Bob Jaffe for his suggestion to use wave packets in the discussion of relativistic corrections.

## A. Nonrelativistic Limit

In the nonrelativistic limit, where  $\frac{E_{\vec{p}} + E_{\vec{p}'}}{2\sqrt{E_{\vec{p}}E_{\vec{p}'}}} = 1$  and  $q^2 = -\vec{q}^2$  one can pull the form factor out of the integral in Eq. (2.4), yielding

$$\mathcal{F}_\psi(\vec{q}) = F(-\vec{q}^2) \int d^3p \Psi^*(\vec{p} + \vec{q}) \Psi(\vec{p}). \quad (2.5)$$

Finally, by using a wave packet  $\Psi$  that is very broad in momentum space (i.e. localized in position space!) the dependence of the overlap integral  $\int d^3p \Psi^*(\vec{p} + \vec{q}) \Psi(\vec{p})$  on  $\vec{q}$  is much weaker than the dependence of the form factor  $F(-\vec{q}^2)$ . For such a wave packet Eq. (2.5) one thus finds

$$\mathcal{F}_\Psi(\vec{q}) = F(-\vec{q}^2) \int d^3p |\Psi(\vec{p})|^2 = F(\vec{q}^2) \quad (2.6)$$

Therefore, in a nonrelativistic theory, <sup>3</sup> as long as one uses a wave packet which is very localized in position space, the Fourier transform of the charge distribution for this wave packet equals the form factor. It is thus legitimate to interpret the form factor as the Fourier transform of the intrinsic charge distribution.

## B. Relativistic Corrections (Rest Frame)

Unfortunately, in a relativistic theory, it is in general not possible to form a wave packet of states whose Fourier transformed charge distribution equals the form factor. In the nonrelativistic case, we used a wave packet that had an arbitrarily small extension in position space. In a relativistic theory, localizing a wave packet to less than its Compton wavelength in size will in general induce various relativistic corrections. This fact is best illustrated by considering the rms radius of this charge distribution [ $2^{nd}$  derivative w.r.t.  $\vec{q}$  in Eq. (2.4)]

Expanding  $\mathcal{F}_\Psi(\vec{q})$  up to, and including  $\mathcal{O}(\vec{q}^2)$ , one finds <sup>4</sup>

$$\begin{aligned} \mathcal{F}_\Psi(\vec{q}) &= 1 - \frac{R^2}{6} \vec{q}^2 - \frac{R^2}{6} \int d^3p |\Psi(\vec{p})|^2 \frac{(\vec{q} \cdot \vec{p})^2}{E_{\vec{p}}^2} \\ &\quad + 1 \int d^3p \left| \vec{q} \cdot \vec{\nabla} \Psi(\vec{p}) \right|^2 - \frac{1}{8} \int d^3p |\Psi(\vec{p})|^2 \frac{(\vec{q} \cdot \vec{p})^2}{E_{\vec{p}}^4}, \end{aligned} \quad (2.7)$$

where  $R^2$  is defined through the slope of  $F(q^2) = 1 + \frac{R^2}{6} q^2 + \mathcal{O}(q^4)$ . In addition to the contribution from the

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<sup>3</sup>Actually, ‘nonrelativistic’ is necessary here only to the extent that the momentum transfer leaves the target nonrelativistic!

<sup>4</sup>We assume here a target with unit charge. The generalization to other values for the charge is straightforward.

intrinsic size and the contribution from the size of the wave packet, one obtains a Lorentz contraction contribution and other relativistic corrections.

Ideally, one would again like to construct a wave packet such that the contribution from the spatial extension of the wave packet, i.e. the term  $\propto \int d^3p \left| \vec{q} \cdot \vec{\nabla} \Psi(\vec{p}) \right|^2$ , is negligible compared to  $R^2 \vec{q}^2$ . Making the corrections due to the extension of the wave packet negligible requires a typical momentum scale in  $\Psi(\vec{p})$  that is much larger than  $\frac{1}{R}$ . This on the other hand leads to contributions from the relativistic corrections in Eq. (2.7) that are at least of the order  $\Delta R^2 \sim \frac{1}{R^2 M^2}$ , which are negligible only if the Compton wavelength of the target is much smaller than its intrinsic size (as defined through the slope of the form factor).

The physics of this result is clear: as soon as one attempts to localize the wave packet to a region smaller than its Compton wavelength, the particle in the wave packet becomes relativistic and relativistic effects, such as Lorentz contraction, are no longer negligible. What this means is that an identification of the slope of the formfactor with the rms-radius of a charge distribution in the rest frame is not unambiguously possible and the best one can achieve is an identification with uncertainties on the order of the Compton wavelength of the target.

### C. Infinite Momentum Frame

In certain frames, such as the IMF (which will be relevant for the application to off-forward parton distribution!), this ambiguity can be avoided. The essential point is that the relativistic corrections are governed by coefficients like  $\frac{\vec{q} \cdot \vec{p}}{E_p^2}$  and  $\frac{q^2}{E_p^2}$ . One way to keep these relativistic coefficients small is to keep  $\vec{q}^2$  and  $\vec{q} \cdot \vec{p}$  finite while sending  $E_{\vec{p}} \rightarrow \infty$ , i.e. by going to the IMF! In the following, let us assume a wave packet such that  $\vec{P} = (0, 0, p_z)$  is the mean momentum of the wave packet and the momentum transfer is purely transverse, i.e.  $\vec{q} = (q_x, q_y, 0)$ . Furthermore, we chose a wave packet that is a plane wave (or very delocalized) in the  $z$ -direction, i.e.  $p_z$  of the wave packet is very sharply peaked around  $P_z$ , and we choose  $P_z$  such that  $P_z \gg M, |\vec{q}|$ . Then the abovementioned corrections due to the wave packet can be made small without leading to large relativistic corrections, which are governed by the expansion parameter  $\frac{\vec{q} \cdot \vec{p}}{E_p^2} \sim \frac{q_\perp}{L_\perp P^2}$ .

In other words, if we consider a wave packet which is localized in the transverse direction only, but a plane wave with very large momentum in the  $z$  direction, then as long as this system is probed with only a transverse momentum transfer, the relativistic corrections to the form factor of this wave packet are governed not by the Compton wavelength but rather by  $\lambda_P \equiv 1/\sqrt{m^2 + p_z^2}$ ,

which can be made arbitrarily small.<sup>5</sup> One thus finds for purely transverse momentum transfers in the IMF

$$\mathcal{F}_\Psi(\vec{q}) = F(\vec{q}^2). \quad (2.8)$$

Physically, this implies that in the IMF one can identify the (Fourier transform of) the charge distribution in the transverse direction (the ‘transverse profile’) with the formfactor without relativistic corrections. Of course, for ordinary form factors, this result is not very important since the IMF is not a natural frame for physical interpretation of the form factor. However, the analogous result will be crucial when we analyze (off forward) parton distributions for which the natural frame *is* the IMF.

### III. OFF FORWARD PARTON DISTRIBUTIONS

Consider a wave packet  $|\Psi\rangle$  which is chosen such that it has a sharp longitudinal momentum  $p_z$ , but whose position is a localized wave packet in the transverse direction

$$|\Psi\rangle = \int \frac{d^2 p_\perp}{\sqrt{2E_{\vec{p}}(2\pi)^2}} \Psi(\vec{p}_\perp) |\vec{p}\rangle \quad (3.1)$$

Clearly,

$$f_\Psi(x, \vec{b}_\perp) \equiv \int \frac{dx^-}{4\pi} \langle \Psi | \bar{\psi}(\vec{b}_\perp, 0) \gamma^+ \psi(\vec{b}_\perp, x^-) | \Psi \rangle e^{ix p^+ x^-}, \quad (3.2)$$

describes the probability to find partons with momentum fraction  $x$  at transverse (position) coordinate  $\vec{x}_\perp$  in this wave packet. What we will show in the following is that  $f_\Psi(x, \vec{b}_\perp)$  can be related to off-forward parton distribution functions with  $\zeta = 0$ .

Using Eq. (3.1), one finds

$$\begin{aligned} \mathcal{F}_\Psi(x, \vec{q}_\perp) &\equiv \int d^2 q_\perp e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} f_\Psi(x, \vec{b}_\perp) \\ &= \int \frac{d^2 p_\perp \Psi^*(\vec{p}'_\perp) \Psi(\vec{p}_\perp)}{\sqrt{2E_{\vec{p}} 2E_{\vec{p}'}}} \times \\ &\quad \int dx^- e^{ix p^+ x^-} \langle p' | \bar{\psi}(0, \vec{0}_\perp) \psi(x^-, \vec{0}_\perp) | p \rangle \\ &= \int \frac{d^2 p_\perp \Psi^*(\vec{p}'_\perp) \Psi(\vec{p}_\perp)}{\sqrt{2E_{\vec{p}} 2E_{\vec{p}'}}} f_\zeta(x, q^2). \end{aligned} \quad (3.3)$$

where  $\vec{p}'_\perp = \vec{p}_\perp + \vec{q}_\perp$  and  $p'_z = p_z$ , i.e.  $\zeta = 0$ .

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<sup>5</sup>Note that this result is reminiscent of the result that in the infinite momentum frame, for purely transverse momentum transfer, only terms that are diagonal in Fock space contribute to the matrix elements of the (‘good’) current [8].

### A. Nonrelativistic Limit

Again we start by investigating the nonrelativistic limit first, where one finds  $E_{\vec{p}} \approx E_{\vec{p}'} \approx m$  and therefore also  $q^2 \approx -\vec{q}_\perp^2$ . As a result, Eq. (3.3) simplifies, yielding

$$\mathcal{F}_\Psi(x, \vec{q}_\perp) = f(x, -\vec{q}_\perp^2) \int \frac{d^2 p_\perp \Psi^*(\vec{p}'_\perp) \Psi(\vec{p}_\perp)}{2m}. \quad (3.4)$$

In order to proceed further, we choose a wave-packet that is very localized in transverse position space. Specifically, we choose a packet whose width in transverse momentum space is much larger than a typical QCD scale. That way, the dependence of the integrand in Eq. (3.4) on  $\vec{q}_\perp$  is mostly due to the matrix element and not due to the wave packet  $\Psi$ . Therefore, by making the wave packet very localized in position space one obtains

$$\mathcal{F}_\Psi(x, \vec{q}_\perp) = f(x, -\vec{q}_\perp^2), \quad (3.5)$$

and, just as it was the case for the form factor, it is thus legitimate to identify the Fourier transform of the  $\zeta = 0$  OFPD with respect to  $\vec{q}_\perp$  with the impact parameter dependence of the parton distribution in a very localized localized wave packet, i.e. with the with the impact parameter dependence of the parton distribution in the target particle itself.

### B. Infinite Momentum Frame

In an arbitrary frame, e.g. the rest frame, relativistic corrections also spoil the above identification of (Fourier transforms of) the impact parameter dependence of parton distributions with OFPD at  $\zeta = 0$ . Similarly to the relativistic corrections for form factors, the above identification becomes ambiguous when one looks at scales smaller than the Compton wavelength of the target.

However, since the natural frame to think about (off forward) parton distributions is the IMF, we will skip details about relativistic corrections in the rest frame and proceed immediately to the IMF. The crucial steps are as follows:

- In Eq. (3.3), we choose a wave packet  $\Psi(\vec{p}_\perp)$  whose typical momentum scale  $\lambda_\Psi$  is much smaller than  $\sqrt{m^2 + p_z^2}$  yet much larger than the expected  $\vec{q}^2$  dependence of  $f(x, -\vec{q}_\perp^2)$ , which should be on the order of  $\Lambda_{QCD}$ .
- we consider only momentum transfers that are smaller than  $\lambda_\Psi$ , i.e. we only probe the target with

Of course, satisfying these requirements simultaneously is only possible for  $p_z \gg m$ .

For a wave packet satisfying the above requirements, it is clear that one can approximate  $E_{\vec{p}} \approx E_{\vec{p}'} \approx |p_z|$ , as well as  $q^2 = -\vec{q}_\perp^2$  in Eq. (3.3), yielding

$$\begin{aligned} \mathcal{F}_\Psi(x, \vec{q}_\perp) &= f(x, -\vec{q}_\perp^2) \frac{1}{2|p_z|} \int d^2 p_\perp \Psi^*(\vec{p}'_\perp) \Psi(\vec{p}_\perp) \\ &= f(x, -\vec{q}_\perp^2) \frac{1}{2|p_z|}, \end{aligned} \quad (3.6)$$

where in the last step we used the fact that we had chosen a very localized wave packet, i.e.

$$\int d^2 p_\perp \Psi^*(\vec{p}_\perp + \vec{q}_\perp) \Psi(\vec{p}_\perp) \approx \int d^2 p_\perp \Psi^*(\vec{p}_\perp) \Psi(\vec{p}_\perp) = 1 \quad (3.7)$$

for  $\vec{q}_\perp^2 = \mathcal{O}(\Lambda_{QCD}^2)$ . In the previous section we had argued that Eqs. (2.6) and (2.8) justify to identify the elastic form factor  $F(q^2)$  with the Fourier transform of the charge distribution in the rest frame (nonrelativistic) and the transverse charge distribution in the infinite momentum frame respectively. In the same vein, Eqs. (3.4) and (3.6) justify to identify  $f_{\zeta=0}(x, t)$  with the Fourier transform of (impact parameter dependent) parton distribution functions with respect to the impact parameter.

Note that, while it would seem unnatural to identify the elastic form factor with something defined in the IMF, the natural frame to think about parton distribution functions (forward and off-forward) is the IMF. Therefore, the fact that Eq. (3.6) is free of relativistic corrections only in the IMF, does not represent a serious restriction at all.

## IV. $Q^2$ EVOLUTION

Throughout this paper we have suppressed the dependence of the parton distributions on the momentum scale  $Q^2$ . Obviously, because of scaling violations, all parton distributions involved depend on  $Q^2$  as well, e.g.  $f(x, \vec{b}_\perp)$  should be replaced by  $f(x, \vec{b}_\perp, Q^2)$  and  $f(x, -\vec{q}_\perp^2)$  should be replaced by  $f(x, -\vec{q}_\perp^2, Q^2)$ .

Fortunately, it is rather straightforward to generalize our results to take  $Q^2$  evolution into account since the  $Q^2$  evolution Eqs. for OFPDs reduce to the usual DGLAP equations for  $\zeta = 0$  [1,2]. Of course, although all parton distributions that enter the DGLAP equations for OFPDs depend on the invariant momentum transfer  $t$ , the evolution equations themselves are impact parameter independent.

Likewise, the impact parameter dependent parton distributions evolve according to the standard DGLAP equations as well in the sense that the same DGLAP equation applies to each  $\vec{b}_\perp$  and different  $\vec{b}_\perp$  do not mix under DGLAP evolution. To see this, one can use translational invariance to shift the  $\vec{b}_\perp$ -dependence on the r.h.s. of Eq. (3.2) from the operator to the state, i.e. instead of measuring the correlator  $\bar{\psi}(\vec{b}_\perp, 0) \gamma^+ \psi(\vec{b}_\perp, x^-)$  in a wave packet centered around  $\vec{0}_\perp$  one can equivalently measure the correlator  $\bar{\psi}(\vec{0}_\perp, 0) \gamma^+ \psi(\vec{0}_\perp, x^-)$  in a wave packet centered around  $-\vec{b}_\perp$ .

Combining these observations it is thus trivial to see that the identification of impact parameter dependent parton distributions with Fourier transforms of  $f(x, -\vec{q}_\perp^2)$  w.r.t  $\vec{q}$  is preserved under QCD evolution in the sense that

$$f(x_{Bj}, \vec{b}_\perp, Q^2) = \int \frac{d^2 q_\perp}{2\pi} e^{i\vec{q}_\perp \cdot \vec{b}_\perp} f_{\zeta=0}(x_{Bj}, -\vec{q}_\perp^2, Q^2) \quad (4.1)$$

is valid for all  $Q^2$  (as long as  $Q^2$  is large enough for DGLAP to be applicable).

## V. SUMMARY AND DISCUSSION

Off-forward parton distributions at  $\zeta = 0$  allow a simultaneous measurement of the light-cone momentum and transverse position (impact parameter!) distribution of partons in a hadron:

$$f(x_{Bj}, \vec{b}_\perp) = \int \frac{d^2 q_\perp}{2\pi} e^{i\vec{q}_\perp \cdot \vec{b}_\perp} f_{\zeta=0}(x_{Bj}, -\vec{q}_\perp^2). \quad (5.1)$$

This fundamental observation is strictly true in the IMF, but receives relativistic corrections in other frames. Those corrections are of the same nature as the relativistic corrections that spoil the identification of the charge form factor with the Fourier transform of a charge distribution for systems where the Compton wavelength is of the same order as the size, i.e.  $MR = \mathcal{O}(1)$ , or larger. Of course in nonrelativistic systems, the identification of  $f_{\zeta=0}(x_{Bj}, \vec{q}_\perp^2)$  with the Fourier transform of the longitudinal momentum/transverse position distribution function is also strictly true.

Moreover, although we restricted our discussion of spin independent parton distribution functions, it should be clear that our result generalizes to spin dependent distribution as well.

While these result is not so much of importance for exact calculations of off-forward distribution functions (for example within the framework of lattice QCD), the main application of our result lies both more within the areas modeling, phenomenology as well as the physical interpretation of experimental and numerical (lattice) data.

However, the most important application is using experimental (or numerical) data on the  $t$  dependence to learn how parton distributions depend on the impact parameter. For example, by considering the slope of  $f_\zeta(x, t)$  w.r.t.  $t$  at  $t = \zeta = 0$  one obtains the parton distribution weighted by the impact parameter squared and thus the ‘outer’ region of the target hadron gets more strongly emphasized. A precise measurement of this slope could thus reveal important information in the transverse distribution of partons within hadrons, which could also help to distinguish surface effects from bulk effects in nucleons and nuclei.

More specific applications, should also include extending models for conventional parton distribution functions to off-forward distributions at  $\zeta \rightarrow 0$ . However, providing explicit examples for this is beyond the scope of this paper.

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